LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION - MATHEMATICS

FIRST SEMESTER – NOVEMBER 2015

MT 1502 - ALGEBRA AND CALCULUS - I

Date : 04/11/2015   Dept. No.   Max. : 100 Marks
Time : 01:00-04:00

PART – A

Answer ALL the questions: **(10 x 2 = 20 marks)**

1. Find the nth derivative of $\sin(ax+b)$.

2. Show that, in the curve $r = a_{"}$ the polar sub tangent varies as the squares of the radius vector and the polar subnormal is constant.

3. Write the condition for the maxima and minima of functions of two variables.

4. Write the steps used in Lagrange's method of undetermined multiples.

5. Write down the roots of equation $ax^3 - bx^2 + cx + d = 0$ in geometric progression.

6. Write down the Cartesian formula for the radius of curvature.

7. Find the sum of the roots of the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$

8. Form a quadratic equation having $3 - \sqrt{-2}$ as a root.

9. Define evolute.

10. Remove the fractional coefficient from the equation $x^3 - 18x^2 + \frac{1}{4}x - \frac{1}{3} = 0.$.

PART – B

Answer any FIVE questions: **(5 x 8 = 40 marks)**

11. If $y = \sin\left(m\sin^{-1} x\right)$ prove that $\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$.

12. Find the angle at which the radius vector cuts the curve $\dfrac{l}{r} = 1 + e\cos_{"}$.

13. Find the maximum or minimum values of $2\left(x^2 - y^2\right) - x^4 + y^4$.

14. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1,1)$.

15. Find the asymptotes of $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$.

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16. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that one of the root is $1 - \sqrt{5}$.

17. Find $\dfrac{1}{r^5} + \dfrac{1}{s^5} + \dfrac{1}{x^5}$ where r, s, x are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.

18. Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

PART – C

Answer any TWO questions: **(2 x 20 = 40 marks)**

19. (i) If $y = a\cos(\log x) + b\sin(\log x),$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$

   (ii) If $u = a^3 x^2 + b^3 y^2 + c^3 z^2$ where $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$, find the minimum value of u.   (10+10)

20. Show that the evolute of the cycloid $x = a(\theta - \sin\theta); y = a(1 - \cos\theta)$ is another cycloid.

21. (i) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.

   (ii) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.   (10+10)

22. (i) Using Horner's method find the real root of the equation $x^3 - 3x + 1 = 0.$ Calculate it to two

   places of decimals.

   (ii) Increase the roots of the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ by 3.   (14+6)

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